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Several Modes for Assessment Efficiency Decision Making Unit in Data Envelopment Analysis with Integer Data

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Abstract

The purpose of this study is to use integer data in the DEA model. In the first stage, we introduced a radial model in which the image was an integer point. Next, we applied this concept to non-radial models. Finally we used a numerical example with the integer data which indicated the importance of this concept.

Keyword: DEA, Integer efficiency, Integer additive, Integer SBM, Integer data.

Introduction

Data envelopment analysis (DEA) is a mathematical programming approach for evaluating the relative efficiency of decision making units (DMUs) with multiple inputs and multiple outputs, where every unit characterized by a pair of non-negative $(x_j, y_j) \in R^{m+s}$ (j=1,..., n) for real input and output vectors (Cooper et al., 2003; Charnels et al., 1978). The traditional models of DEA, the data real amount, but in some cases the data has been limited to the integer value. First Lozano and Villa (2006) with an example highlighted the difference between the two scenarios (Integer & Real) and with the integer constraints on the model calculated efficiency units. Next the Kuosmanen and Kazemi Matin (2009) introduced the basic principles and proposed a produce possibility set (PPS) and an integer model to calculate the efficiency integer units. In the following, Kazemi-Matin and Kuosmanen (2009) with regard to assumption of variable returns to scale and extend principle developed their previous work. In this paper, was provides a radial integer model to calculate the performance units, integer additive model and then non radial integer model to calculate the unit's efficiency.

The remaining structure of this study is organized as follows: In section 2 we provided prerequisite of DEA by the integer data. In section 3, we proposed a radial model and an additive model with integer data and then a non-radial model to calculate the efficiency of integer units. In section 4, an example from real world is given to address the importance of topic. Section 5 contains conclusions.

Prerequisites

Supposes are given n decision making units as $\{(x_j, y_j) | j = 1, \dots, n\}$, that $x_j = (x_{1j}, \dots, x_{mj})^T$ and $y_j = (y_{1j}, \dots, y_{sj})^T$ are the vectors of non-negative input and output of j units, also $X = [x_1, \dots, x_n]^T$ and $Y = [y_1, \dots, y_n]^T$ are the input and output matrices, respectively. In standard DEA models we assumed that all data to be real values. But the worlds around us, some data has been limited to be integer. First Lozano and Villa (2007) due to these differences and introducing integer constraints in the context of DEA, introduced a mixed integer linear programming model for performance units. After them Kuosmanen and Kazemi Matin (2009) by introducing axioms:

1. Envelopment: $(x_j, y_j) \in T \quad \forall j = 1, \dots, n$
2. Natural disposability: $(x, y) \in T \ , \ (u, v) \ \in Z_+^{m+s}, y \geq v \ , (x + u, y - v) \in T$

Natural convexity:

$(x,y),(x',y') \in T, (x'',y'') = \lambda(x,y) + (1-\lambda)(x',y'), \lambda \in [0,1], (x'',y'') \in Z_+^{m+s} \rightarrow (x'',y'') \in T$

1. Natural divisibility: $(x,y) \in T, \exists\, \lambda \in [0,1]; (\lambda x, \lambda y) \in Z_+^{m+s} \rightarrow (\lambda x, \lambda y) \in T$

2. Natural augment ability: $(x,y) \in T, \exists\, \lambda \geq 1, (\lambda x, \lambda y) \in Z_+^{m+s} \rightarrow (\lambda x, \lambda y) \in T$

They constructed an integer production possibility set (PPS): where if the technology involves the inclusion of observations and principles 1 and 2, the production possibility set is:

$$T_{VRS}^{IDEA} = \left\{(x,y) \in Z_+^{m+s} \middle| x \geq \sum_{j=1}^{n} \lambda_j x_j \;;\; y \leq \sum_{j=1}^{n} \lambda_j y_j \;;\; \sum_{j=1}^{n} \lambda_j = 1 \;;\; \lambda_j \geq 0 \; j = 1, \ldots, n\right\}$$

And if in addition to the above principles, principle 3 is satisfied, the production possibility set is:

$$T_{NIRS}^{IDEA} = \left\{(x,y) \in Z_+^{m+s} \middle| (x,y) \in T_{VRS}^{IDEA} \;;\; \sum_{j=1}^{n} \lambda_j \leq 1\right\}$$

Also, if in addition to the envelopment observations and principles 1 and 2 is hold principle 4, the production possibility set will be:

$$T_{NDRS}^{IDEA} = \left\{(x,y) \in Z_+^{m+s} \middle| (x,y) \in T_{VRS}^{IDEA} \;;\; \sum_{j=1}^{n} \lambda_j \geq 1 \right\}$$

And if principles inclusion of all observations, 1, 2, 3 and 4 hold, the production possibility set is:

$$T_{CRS}^{IDEA} = \left\{(x,y) \in Z_+^{m+s} \middle| x \geq \sum_{j=1}^{n} \lambda_j x_j \;;\; y \leq \sum_{j=1}^{n} \lambda_j y_j \;;\; \lambda_j \geq 0\right\}$$

Kuosmanen and Kazemi-Matin (2009) presented the flowing radial model for performance evaluation:

Model 1

$$\min \theta - \varepsilon\left(\sum_{i=1}^{m} s_i^- + \sum_{r=1}^{s} s_r^+ + \sum_{r=1}^{s} s_i^I\right)$$

$$\sum_{j=1}^{n} \lambda_j x_{ij} + s_i^- = \theta x_{io} \quad i \in I - I_I$$

$$\sum_{j=1}^{n} \lambda_j y_{rj} + s_r^+ = y_{ro} \quad r \in O$$

$$\sum_{i=1}^{m} \lambda_j x_{ij} + s_i^- = \hat{x}_i \quad i \in I_I$$

$$\hat{x}_i = \theta x_{io} - s_i^I \quad i \in I_I$$

$$\hat{x}_i \in Z_+ \quad i \in I_I$$

$$\lambda_j \geq 0$$

$$s_i^- \geq 0, \; s_i^I \geq 0$$

$$s_r^+ \geq 0$$

Concepts Of the DEA by integer data

The purpose of this section is to provide several models for evaluating the efficiency of decision making units that ensure the target point is integer. Assume that give n DMUs are homogeneous with the data of non-negative integers. Each one consumes m integer inputs for production s integer outputs. If vector input and output unit j (j=1,..., n) donate with symbols $\boldsymbol{x}_j = (x_{1j}, \ldots, x_{mj})^T \in Z_+^m$ and $\boldsymbol{y}_j = (y_{1j}, \ldots, y_{sj})^T \in Z_+^s$, in this case consider the following integer radial model for evaluation unit (x_o, y_o) :

Model 2

$$Min \;\; \theta - \varepsilon\,[1^T(S^+ - q) + 1^T(S^- - p)]$$

s.t

$$X\lambda + S^- = \theta x_o$$

$$Y\lambda + S^{+} = y_o$$
$$y_o + S^{+} = T + q$$
$$\theta x_o - S^{-} = k + p$$
$$S^{+} \geq o \ , q \geq o$$
$$S^{-} \geq o \ , p \geq o$$
$$k \geq o \ , \ T \geq o$$
$$\lambda \geq o$$
$$(k, T) \in Z^{m+s}$$
$$\begin{bmatrix} 1\lambda = 1 \\ 1\lambda \geq 1 \\ 1\lambda \leq 1 \end{bmatrix}$$

Such that information in the brackets ate for the assumptions of variable return to scale, non-descending and non-increasing respectively.
In the above model:

$$(\theta, \lambda, S^{-}, S^{+}, p, q, k, t) = (1, e_o, o, o, o, o, x_o, y_o)$$

One integer feasible solution and ε is a very small number of non-Archimedes. Therefore, we must have in the optimal solution p=o, q=o, then the target point is integer.

Definition 1

DMU_o is CCR integer efficient, if optimal value of objective function model 2 corresponds to assumption constant returns to scale of 1.

Definition 2

DMU_o is BCC integer efficient, if optimal value of objective function model 1 corresponds to assumption of variable returns to scale of 1.
Also, integer additive model for evaluation (x_o, y_o) is presented as follows:

Max $1S^{-} + 1S^{+}$
s.t
$$(x_o - S^{-}, y_o + S^{+}) \in T_t$$
$$(S^{-}, S^{+}) \in Z_{+}^{m+s}$$

Where $t = \{VRS, NIRS, NDRS, CRS\}$
If consider the assumption of variable returns to scale (VRS), in this case the model is converted to:

Model 3

Max $1S^{-} + 1S^{+}$
s.t.
$$X\lambda + S^{-} \leq x_o$$
$$Y\lambda + S^{+} \geq y_o$$
$$1\lambda = 1$$
$$(S^{-}, S^{+}) \in Z_{+}^{m+s}$$

Definition 3

DMU_o is BCC integer efficient, if the optimal value of model 3 is equal to zero.
According to the additive model which has been introduced, the SBM model for integer data is presented as follows:

Model 4

Min $\rho = 1 - \frac{1}{m} \sum_{i=1}^{n} S_i^{-} / x_{io}$

s.t.

$$X\lambda + S^- \leq x_o$$
$$Y\lambda + S^+ \geq y_o$$
$$(S^-, S^+) \in Z_+^{m+s}$$
$$\lambda \geq o$$
$$\begin{bmatrix} 1\lambda = 1 \\ 1\lambda \geq 1 \\ 1\lambda \leq 1 \end{bmatrix}$$

The above model is ISBM model, where inside brackets assuming variable returns to scale, non-decreasing return to scale and non-increasing returns to scale respectively: which given efficiency value and image point is:

$$(x_o - S^{-*}, y_o - S^{+*})$$

That is an integer point.

Note: Assume that $(\theta^*, \lambda^*, S^{-*}, S^{+*}, K^*, t^*, q^*, p^*)$ is an optimum solution of model 2: in this case we have:

$$x_o = X\lambda^* + S^{-*}\ (1 - \theta^*)x_o$$
$$y_o = Y\lambda^* - S^{+*}$$

In this case, the $(S^{-*} + (1 - \theta^*)x_o\ , S^{+*})$ may not be all components are integer, so may not be a feasible solution for ISBM model, we cannot conclude as real state: $\rho^* \leq \theta^*$.

Numerical example

One of the major applications of DEA is to evaluate the various branches of banks that have been discussed in many research works. In our example given the 25 branches of bank is one of the provinces of Iran contain 4 inputs and 3 outputs. To evaluate bank branches, the inputs are:

1. Costs may be receivable
2. Personnel costs
3. Capital costs
4. Equipment costs of branch.

And outputs are:

1. Species incomes
2. Species guarantees
3. Species bank convenience.

Where input data and output data is presented in table 1.

Table 1. Input and output data.

DMU	Input1	Input2	Input3	Input4	Output1	Output2	Output3
1	3422	4012	4353	3525	8921	5842	7512
2	3899	4316	4528	4656	5618	7343	6200
3	3478	4802	3874	3270	5468	5698	5102
4	4236	3145	3334	4504	8423	9821	8821
5	4821	3910	4140	4756	9181	6879	7305
6	4110	3487	3546	3123	6752	6521	9700
7	3980	4512	3487	3123	6752	6521	7546
8	4741	4231	4132	4523	6458	5600	9000
9	3422	3568	3961	3999	8010	5000	5887
10	4802	3454	4215	3792	7039	6015	5642
11	3050	4988	3971	4832	9253	8433	5897
12	3645	3753	4270	4219	5812	4999	6658
13	4910	3999	4190	3190	7314	5488	4599
14	4720	3491	3564	4802	6541	8324	7895
15	3879	4258	3500	3613	8741	9541	7291
16	4512	4908	4208	3692	9718	9291	8102
17	3691	4325	3222	5000	5642	7518	9941
18	4321	3867	3224	4003	10000	6465	9249

19	3784	3312	3989	3722	9758	6128	6709
20	3465	4657	3874	4918	7302	7312	7032
21	4410	4415	4632	3558	8821	6218	8245
22	3333	3720	4228	3292	5912	7324	8914
23	3784	4666	4220	4818	7543	8499	9214
24	4825	4777	3890	4391	6100	7666	4521
25	4325	3525	4471	3517	7415	7946	7415

As we can see, inputs and outputs are the entire integer. Then the use integer models introduced in this paper to evaluate the performance of these banks. In order to compare with the previous methods in the literature integer model, also it has been used by Kuosmanen and Kazemi Matin (2009).For the evaluation: they used proposed radial model of, Kuosmanen and Kazemi Matin (2009) and ISBM model. To solve the model, we used the Lingo 8 software. Information to assess the performance of the methods is given in table 2. In table 2, θ_K is efficiency of the proposed model by Kuosmanen and Kazemi Matin (2009). Also θ_{IS} the efficiency of proposed radial model and θ_{ISBM} is efficiency in ISBM model. For all three models, we have assumed constant returns to scale. As you can see from the table, for each of the three models, units 1, 4, 6, 11, 15, 16, 17, 18, 19 and 22 are integers efficient. As explained in the previous section, we cannot guarantee the efficiency of the ISBM model is more than efficiency of radial model and this better than is clear in this instance, where except unit 21 efficiency ISBM model not lower than the efficiency of proposed models by Kuosmanen and Kazemi Matin (2009).

Table 2. Efficiency scores.

DMU	ƟK	ƟIS	ƟISBM
1	1	1	1
2	0.765	0.765	0.945
3	0.716	0.716	0.940
4	1	1	1
5	0.845	0.845	1
6	1	1	0.972
7	0.951	0.951	0.986
8	0.796	0.796	0.961
9	0.890	0.890	0.970
10	0.811	0.811	0.957
11	1	1	1
12	0.758	0.758	0.946
13	0.871	0.871	0.955
14	0.824	0.824	0.966
15	1	1	1
16	1	1	1
17	1	1	1
18	1	1	1
19	1	1	1
20	0.894	0.894	0.960
21	0.978	0.978	0.972
22	1	1	1
23	0.988	0.988	0.991
24	0.705	0.705	0.946
25	0.958	0.958	0.975

Conclusion

In this paper, firstly a radial model for calculating the efficiency of DMUs with the integer inputs and outputs has been presented. Also an integer additive model and the ISBM model, that is a generalization of the SBM model to the integer data, have been presented. Finally, an example of real world has been solved to demonstrate the importance of topic. The results revealed that, the models proposed in this paper were comparable with the radial model presented by the Kuosmanen and Kazemi Matin (2009) model were almost the same. In this paper we assumed that all inputs and outputs are integer but we can also generalize the model for a scenario that the inputs and outputs be both integer and real.

References

Charnels A, Cooper WW, Rhodes E, 1978. Measuring the efficiencies of DMUs. European Journal of Operational Research. 2(6): 429-444.

Cooper WW, Seaford LM, Tone K, 2003. Data envelopment analysis: a comprehensive text with models, applications, references, and DEA-solver software. 4rd Edn. Boston: Kluwer Academic Publishers.

Kazemi Matin R, Kuosmanen T, 2009. Theory of integer-valued data envelopment analysis under alternative returns to scale axioms. Omega. 37: 988-995.

Kuosmanen T, Kazemi Matin R, 2009. Theory of integer valued data envelopment analysis. European Journal of Operational Research. 192: 658-667.

Lozano S, Villa G, 2006. Data envelopment analysis of integer-valued inputs and outputs. Computers and Operations Research. 33(10): 3004-3014.

Lozano S, Villa G (2007). Integer DEA models. In: Zhu J, Cook WD, Modeling data irregularities and structural complexities in data envelopment analysis. New York: Springer.